

Modeling Time-Varying Dependencies between Positive-Valued High-Frequency Time Series

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Abstract Multiplicative error models (MEM) became a standard tool for modeling conditional durations of intraday transactions, realized volatilities and trading volumes. The parametric estimation of the corresponding multivariate model, the so-called vector MEM (VMEM), requires a specification of the joint error term distribution, which is due to the lack of multivariate distribution functions on \mathbb{R}_+^d defined via a copula. Maximum likelihood estimation is based on the assumption of constant copula parameters and therefore, leads to invalid inference, if the dependence exhibits time variations or structural breaks. Hence, we suggest to test for time-varying dependence by calibrating a time-varying copula model and to re-estimate the VMEM based on identified intervals of homogenous dependence. This paper summarizes the important aspects of (V)MEM, its estimation and a sequential test for changes in the dependence structure. The techniques are applied in an empirical example.

Keywords: vector multiplicative error model, copula, time-varying copula, high-frequency data

JEL classification: C32, C51

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1 Multiplicative error models

MEMs are frequently applied to describe autocorrelated positive-valued processes. The multiplicative structure became popular in the context of (G)ARCH models, see [8, 2]. [9] adopted this multiplicative approach to analyze the conditional duration of irregularly spaced financial transaction data under the assumption that the error term follows an Exponential or Weibull distribution. This extends directly to MEMs, when other positive-valued random variables such as, e.g., trading volumes, are considered. As stressed by [9], a joint model including volumes, transaction prices and time variations in liquidity gives a better understanding of the fundamental mechanisms of stock markets than individual univariate analyses.

1.1 Univariate MEM

Let x_i be a non-negative univariate time series, with time index $i = 1, \dots, n$. The univariate MEM is defined as

$$\begin{aligned} x_i &= \mu_i \varepsilon_i \\ \mu_i &\stackrel{\text{def}}{=} \mathbb{E}(x_i | \mathcal{F}_{i-1}; \xi), \end{aligned} \quad (1)$$

where ξ denotes an m -dimensional vector of parameters and the scale factor μ_i is assumed to be measurable with respect to the information set \mathcal{F}_{i-1} . Furthermore, assume that ε_i follows an iid process with $\mathbb{E}(\varepsilon_i) = 1$ and density $f(\cdot)$. The conditional mean can be specified in several ways, e.g.,

$$\mu_i = \omega + \sum_{j=1}^P \alpha_j x_{i-j} + \sum_{j=1}^Q \beta_j \mu_{i-j}, \quad (2)$$

with $\omega \geq 0$, $\alpha_j \geq 0$ and $\beta_j \geq 0$, $\forall j$, $\xi = (\omega, \alpha_1, \dots, \alpha_P, \beta_1, \dots, \beta_Q)^\top$. Based on the filters $\phi(L) = \sum_{j=1}^R \phi_j L^j = \sum_{j=1}^R (\alpha_j + \beta_j) L^j$, $\beta(L) = \sum_{j=1}^Q \beta_j L^j$ and the martingale difference series $\eta_i = x_i - \mu_i$, (2) can be transformed to an ARMA(R, Q) model

$$x_i = \omega + \phi(L)x_i + \{1 - \beta(L)\} \eta_i, \quad (3)$$

where $R = \max(P, Q)$ and L denotes the lag operator with $L^j x_i = x_{i-j}$. According to standard time series arguments, (3) is guaranteed to be weakly stationary, if $\sum_{j=1}^P \alpha_j + \sum_{j=1}^Q \beta_j < 1$. Given the above set of assumptions, we implicitly assume an exponential decay of the autocorrelation function $\rho(\cdot)$, i.e., $\lim_{l \rightarrow \infty} \sum_{j=-l}^l |\rho(j)| < \infty$. However, in case of financial high-frequency data this assumption is often not fulfilled.

As such data typically reveal long memory, we provide a short review of the fractionally integrated MEM (FIMEM), which allows the autocorrelation function

of the underlying random variable to decay hyperbolically. Formally, x_i exhibits long memory if $\lim_{l \rightarrow \infty} \sum_{j=-l}^l |\rho(j)| = \infty$. Following [1], [14] specifies the FIMEM in the context of conditional durations by introducing the fractional difference operator $(1-L)^\delta$ to equation (3), i.e.,

$$\{1 - \phi(L)\} (1-L)^\delta x_i = \omega + \{1 - \beta(L)\} \eta_i, \quad (4)$$

with $\delta \in [0, 1]$ the fractional integration parameter. [13] defines the fractional difference operator by a binomial series:

$$(1-L)^\delta = \sum_{j=0}^{\infty} \binom{\delta}{j} (-1)^j L^j = \sum_{j=0}^{\infty} \pi_j L^j. \quad (5)$$

Substituting the martingale difference series defined above in (4) leads to

$$\begin{aligned} \{1 - \beta(L)\} \mu_i &= \omega + \left[1 - \beta(L) - \{1 - \phi(L)\} (1-L)^\delta\right] x_i \\ \mu_i &= \omega \{1 - \beta(1)\}^{-1} + \lambda(L) x_i, \end{aligned} \quad (6)$$

where $\lambda(L) = \sum_{j=1}^{\infty} \lambda_j L^j$. The linear filter $\lambda(L)$ implies an infinite number of parameter restrictions to guarantee the non-negativity of μ_i , i.e., $\lambda_j \geq 0, \forall j$. As a consequence, in practice the filter $\lambda(L)$ is truncated to a finite number of lags or one needs to apply Theorem 3 of [6] to verify that the combination of parameters of the FIMEM($P; \delta; Q$) are within the feasible parameter space. To emphasize this point, consider the following two extreme examples for which we assume that δ lies within the unit interval, such that $\pi_j < 0$, for $j > 0$. Then, (i) μ_i can become negative although all parameters are greater than zero and (ii) μ_i can be positive almost surely for all i , even though all parameters except δ are negative. Note, that these restrictions play a fundamental role for the validity of forecasts.

The first unconditional moment of x_i is not defined, since the fractional difference operator evaluated at $L = 1$ equals zero. As a result, the FIMEM is not covariance stationary. If the parameters are non-negative and $\sum_{j=1}^P \alpha_j + \sum_{j=1}^Q \beta_j < 1$, then the strict stationarity and ergodicity of the FIMEM can be deduced from the stationarity and ergodicity of the integrated MEM, since the infinite-order representation of (6) is dominated in an absolute value sense by the coefficients of the corresponding integrated MEM, cf. [3, 1]. Alternative covariance stationary long memory MEMs are discussed in [12].

In general, parametric ML estimation of univariate MEMs leads to asymptotically efficient and unbiased estimates, if the distribution of the innovations ε_i is specified correctly. Typical candidates to describe ε_i are the standard Exponential or Weibull distribution, but flexible distributions as the generalized Gamma or F distribution can also be considered. In a standard ML framework for time series models, where $\ell_i(\xi)$, $i = 1, \dots, n$, denotes the i -th contribution to the log likelihood $\ell(\xi) = \sum_{i=1}^n \ell_i(\xi)$, $H_n(\xi) = \sum_{i=1}^n \left\{ \frac{\partial^2}{\partial \xi \partial \xi^\top} \ell_i(\xi) \right\}$ denotes the Hessian matrix and $S_n(\xi) = \sum_{i=1}^n \left\{ \frac{\partial}{\partial \xi} \ell_i(\xi) \frac{\partial}{\partial \xi^\top} \ell_i(\xi) \right\}$ the outer product of scores, the limiting dis-

tribution of the estimator $\hat{\xi}$ is given by

$$\{H_n(\xi)^{-1}S_n(\xi)H_n(\xi)^{-1}\}^{-1/2}\sqrt{n}(\hat{\xi}-\xi)\xrightarrow{\mathcal{L}}N(0_m,I_m), \quad (7)$$

with identity matrix I_m . Statistical inference is based on the finite sample approximation of (7), i.e., the Hessian matrix and the outer score product are replaced by the consistent estimates $H_n(\hat{\xi})$ and $S_n(\hat{\xi})$.

Furthermore, [9] adopts the asymptotic theory of [16] and proposes a quasi-ML setup which leads to consistent estimates for the linear and integrated MEM even if the true error term distribution does not correspond to the assumed standard Exponential distribution. In this case, $\hat{\xi}$ converges under some regularity conditions to the asymptotic distribution of (7) as long as the conditional mean is correctly specified.

1.2 Vector MEM

[5] formalizes the VMEM as

$$x_i = \mu_i \odot \varepsilon_i, \quad (8)$$

where \odot denotes the Hadamard product and $x_i = (x_{i1}, \dots, x_{id})^\top$, $i = 1, \dots, n$, is the vector of positive valued processes. The multivariate scale factor $\mu_i \stackrel{\text{def}}{=} E(x_i | \mathcal{F}_{i-1}; \xi)$ and the vector of error terms ε_i are $(d \times 1)$ vectors. The natural multivariate extension of (6) is given by

$$[I_d - B(L)]\mu_i = \omega + [I_d - B(L) - \{I_d - \Phi(L)\}D]x_i, \quad (9)$$

with $\Phi(L) = A(L) + B(L)$ and A, B being $(d \times d)$ matrices. Short-run effects enter equation (9) through the linear filters $A(L)$ and $B(L)$ and ω denotes the vector of constants. The univariate fractional difference operator from (6) extends to the diagonal matrix $\text{diag}(D) = \{(1-L)^{\delta_1}, \dots, (1-L)^{\delta_d}\}$, which contains the individual fractional difference operators, with $\delta_j \in [0, 1]$, $j = 1, \dots, d$. By this restriction, we exclude deterministic low frequency patterns between the marginal time series. Note that the individual mean equations of (9) collapse to the univariate FIMEM (6), if A and B are diagonal and to the linearly parameterized MEM (2), if additionally $\delta_j = 0$, $j = 1, \dots, d$. Based on the diagonality assumption for A and B the model can be estimated equation by equation and is stationary.

For the full parametric specification of the VMEM we need to define an innovation process ε_i , $i = 1, \dots, n$, which must follow a distribution with only positive probabilities on $\mathbb{R}_+^d = [0, \infty)^d$ and $E(\varepsilon_{ij}) = 1$, $j = 1, \dots, d$. However, the distribution function of a univariate error term process does not have a natural multivariate equivalent. Therefore, the d marginal distributions are coupled together with a copula splitting a multivariate distribution function into its margins and a pure dependence component – the copula. Copulae are introduced in [23] stating that if F is an arbitrary d -dimensional continuous distribution function of the random variables

X_1, \dots, X_d , then the associated copula is unique and defined as a continuous function $C : [0, 1]^d \rightarrow [0, 1]$ which satisfies the equality

$$C(u_1, \dots, u_d) = F\{F_1^{-1}(u_1), \dots, F_d^{-1}(u_d)\}, \quad u_1, \dots, u_d \in [0, 1], \quad (10)$$

where $F_1^{-1}(\cdot), \dots, F_d^{-1}(\cdot)$ are the quantile functions of the continuous marginal distribution functions $F_1(x_1), \dots, F_d(x_d)$. Based on the copula density $c(\cdot, \dots, \cdot; \theta)$, the log likelihood of the VMEM can be written as

$$\begin{aligned} \ell(\theta, \xi, \alpha | \mathcal{F}_{i-1}) &= \sum_{i=1}^n \sum_{j=1}^d [\log [\varepsilon_{ij}(\xi_j) f_j \{\varepsilon_{ij}(\xi_j); \alpha_j\}] - \log x_{ij}] \\ &\quad + \sum_{i=1}^n \log c[F_1\{\varepsilon_{i1}(\xi_1), \alpha_1\}, \dots, F_1\{\varepsilon_{id}(\xi_d), \alpha_d\}; \theta], \end{aligned} \quad (11)$$

with $x_i/\mu_i(\xi) | \mathcal{F}_{i-1} = \varepsilon_i(\xi) | \mathcal{F}_{i-1} \sim C[F_1\{\varepsilon_{i1}(\xi_1), \alpha_1\}, \dots, F_1\{\varepsilon_{id}(\xi_d), \alpha_d\}; \theta]$ having expectation one, where θ denotes the copula-, α the marginal- and ξ the mean-parameters, cf. [5]. Conversely to the Hadamard product, x_i/μ_i denotes element-wise division. The efficient approach to obtain parameter estimates is given by full ML estimation, as the multivariate density function is assumed to be known, i.e., the product of the marginal densities multiplied with the copula density. On the other hand, full ML estimation is difficult to implement even if the induced dependence is non-elliptical. E.g., if we assume a Vine- or hierarchical Archimedean copula (HAC), (see Section 2), the copula density varies with the structure of the underlying copula. Thus, the log likelihood must be optimized for each possible structure and the parameter vector generating the largest log likelihood value is selected as ML estimate. To avoid this computationally intensive method, a two-step procedure similar to [4] can be straightforwardly applied, since (11) can be decomposed into a marginal and a copula part as follows: First, the parameters of the mean equation are estimated to filter the residuals, for which only the information about the marginal distributions is used. Then, the copula is calibrated to the fitted values of the residuals' empirical distribution functions.

Similar to classical risk management applications, where several time-varying models for correlations and copulae are proposed, e.g., [7, 22], time-varying dependence cannot be excluded in our context and consequently, the copula estimated at the second step may contain time variations. Yet, the final target of VMEMs is not to predict, e.g., tail dependencies or risk measures, but to produce forecasts of μ_i , which crucially depend on precise parameter estimates and thus on the complete log likelihood and the most recent data for which the dependence between the variables is constant. Thus, we suggest to re-estimate the parameters of μ_i by maximizing (11) with fixed θ for time intervals at which the copula model calibrated at the second step supports constant dependence.

2 Hierarchical Archimedean Copulae

Among other important families, there exists the class of Archimedean copulae (AC), which (i) permits modeling non-elliptical dependencies, (ii) can describe different types of tail dependencies and (iii) has a closed form expression. Formally, AC are defined through the generator function $\phi_\theta \in \mathcal{L} = \{\phi_\theta : [0; \infty) \rightarrow [0, 1] \mid \phi_\theta(0) = 1, \phi_\theta(\infty) = 0; (-1)^i \phi_\theta^{(i)} \geq 0; i \in \mathbb{N}\}$ and $(-1)^i \phi_\theta^{(i)}(x)$ being non-decreasing and convex on $[0, \infty)$, for $x > 0$, which commonly depends on a single parameter θ , i.e.,

$$C(u_1, \dots, u_d; \theta) = \phi_\theta \{ \phi_\theta^{-1}(u_1) + \dots + \phi_\theta^{-1}(u_d) \}, \quad u_1, \dots, u_d \in [0, 1]. \quad (12)$$

Properties of Archimedean copulae are reviewed and investigated in [18, 15]. [19] discusses generator families depending on two parameters. The restricted dependence structure induced by Archimedean generators is the major disadvantage of d -dimensional ACs, since this assumption is mostly violated in practice.

To permit more flexibility, arguments of an AC can be replaced by further ACs leading to the concept of HAC, which can adopt arbitrary complicated structures denoted by s in the following. The generators of a single HAC, ϕ_j , can come from different generator families. However, if the ϕ_{θ_j} 's come from the same family, the required complete monotonicity of $\phi_{\theta_{j+1}}^{-1} \circ \phi_{\theta_j}$ imposes constraints on the parameters $\theta_1, \dots, \theta_{d-1}$. The flexibility induced by the structure is accompanied by larger amounts of parameters, as each generator composition corresponds to one additional parameter. Sufficient conditions on the generator functions guaranteeing that C is a copula are stated in [17]. It holds that if $\phi_{\theta_j} \in \mathcal{L}$, for $j = 1, \dots, d-1$, and $\phi_{\theta_{j+1}}^{-1} \circ \phi_{\theta_j}$ have completely monotone derivatives, then C is a copula for $d \geq 2$. The major advantage of HACs compared to ACs is the non-exchangeability of the arguments beyond a single node, which is imposed by the structure of a HAC. Similar to the dependence parameters, s is generally unknown and can be regarded as an additional parameter to estimate.

A sequential estimation procedure for HACs is discussed by [20] providing statistical inference for parametric and nonparametric estimated margins. The procedure uses Proposition 1 of [21] stating that HACs can be uniquely reconstructed from marginal distributions and bivariate copula functions. The estimation procedure can be summarized in the following way: at the first step, estimate all binary copula parameters of a specified Archimedean family under the assumption of known marginal distribution functions. Select the largest parameter and fix the binary copula as pseudo-variable. At next steps, assume the estimated margins and sub-copulae from lower levels are known and estimate all binary copula parameters by considering pairs of margins, pairs of pseudo variables and pairs of margins and pseudo variables. Then, choose the largest parameter and fix the corresponding copula as a pseudo variable. This procedure leads a binary approximation of an arbitrary HAC. Let $\varepsilon_i = \{\varepsilon_{i1}, \dots, \varepsilon_{id}\}^\top$ be the sample, $i = 1, \dots, n$, and $\theta = (\theta_1, \dots, \theta_{d-1})^\top$ be the copula parameters ordered from the lowest to the highest hierarchical level.

The multi-stage ML-estimator, $\hat{\theta}$, provides a solution for the following system of equations

$$\begin{aligned} & \left(\frac{\partial \ell_1}{\partial \theta_1}, \dots, \frac{\partial \ell_{d-1}}{\partial \theta_{d-1}} \right)^\top = \mathbf{0}, \quad (13) \\ \text{where } \ell_j &= \sum_{i=1}^n l_j(\epsilon_i), \text{ for } j = 1, \dots, d-1, \\ l_j(\epsilon_i) &= \log \left\{ c \left[\{\hat{F}_m(\epsilon_{im})\}_{m \in s_j}; s_j, \theta_j \right] \prod_{m \in s_j} \hat{f}_m(\epsilon_{im}) \right\} \\ & \text{for } i = 1, \dots, n, \end{aligned}$$

where ℓ denotes the copula part of (11) and s_j contains the indices, which are structured according to the fixed subcopulae (and margins) at lower hierarchical levels.

3 Change point detection

The time intervals for which the parameters of μ_i should be re-estimated are identified by calibrating a time-varying copula. In this context, [11] proposes a framework, which incorporates time-varying HAC parameters θ_i and s_i , and is closely related to the local change point (LCP) procedure applied in [24]. As a detailed discussion of this sophisticated method is beyond the scope of this paper, this section describes only the main ideas of the data driven LCP.

Let θ_i, s_i be the unknown time-varying parameters and structure of the HAC C . Let $I = [i_0 - m, i_0]$ denote an interval with reference point i_0 , $m > 0$ and let $\Delta_I(\theta, s) = \sum_{i \in I} \mathcal{K} \{c(\cdot, \theta_i, s_i), c(\cdot; \theta, s)\}$ be a random quantity, where $\mathcal{K}(\cdot, \cdot)$ denotes the Kullback-Leibler divergence. Furthermore, let $\Delta_I(\theta, s) \leq \Delta$ be the small modeling bias (SMB) condition with $\Delta \geq 0$ and constant parameters θ, s . As $\mathcal{K}(\cdot, \cdot)$ measures the discrepancy between two densities, the data generating process can be well approximated by the local constant copula $C(\cdot; \theta, s)$ on I in the sense of the SMB condition. Based on this condition [11] proposes testing whether a HAC with time-varying parameters and structure can be locally approximated by a HAC with constant parameters and structure.

Under the null hypothesis assume that the SMB condition holds for interval I and parameters $\{\theta, s\}$ and define the set of possible change points \mathcal{T}_I for interval I , which is tested for a single but unknown change point $\tau \in \mathcal{T}_I$. The test hypotheses are formalized as

$$\begin{aligned} H_0 : & \quad \forall \tau \in \mathcal{T}_I, \theta_i = \theta, s_i = s, \forall i \in I = J \cup J^C = [\tau, i_0] \cup [i_0 - m, \tau) \quad (14) \\ H_1 : & \quad \exists \tau \in \mathcal{T}_I, \theta_i = \theta_1, s_i = s_1, \forall i \in J = [\tau, i_0], \\ & \quad \text{and } \theta_i = \theta_2 \neq \theta_1 \text{ or } s_i = s_2 \neq s_1, \forall i \in J^C = [i_0 - m, \tau). \end{aligned}$$

The null hypothesis is rejected, if the likelihood ratio (LR) test statistic

$$T_I = \max_{\tau \in \mathcal{T}_I} \left[\max_{\theta_1, s_1} \{\ell_J(\theta_1, s_1)\} + \max_{\theta_2, s_2} \{\ell_{JC}(\theta_2, s_2)\} - \max_{\theta, s} \{\ell_I(\theta, s)\} \right], \quad (15)$$

exceeds the critical value z_I . In practice, the length of the homogenous interval and the parameters of interest $\{\theta, s\}$ are estimated simultaneously due to their relation through the test statistic. For a well performing choice of the critical value, which is found via a Monte-Carlo simulation from the local parametric model and implicitly defines the significance level of the test statistic T_I , we refer to [24].

4 Empirical analysis

The considered time span of NASDAQ trade data for Apple (AAPL) starts at the January 2nd and ends at December 31th, 2009. Similar to the cleaning of TAQ data sets as, e.g., applied in [12], all non-executed trades, trades with a price smaller or equal to zero and outliers are removed from our tick-by-tick high-frequency data set. To overcome the phenomenon of simultaneous observations, trades with the same time stamp are merged and the corresponding values are aggregated by their median. A cleaned tick-by-tick data set provides information about (i) the price series p_j , (ii) the amount of traded shares s_j and (iii) the time stamp of the trades t_j , $j = 1, \dots, n^*$, where n^* is the number of daily observations. To investigate the relationships between these series, we construct the series of high-low ranges (HL), average volumes (Vol) and the number of trades (NT) on a sampling frequency of 10 min, i.e.,

$$\begin{aligned} \text{HL}_i &= \max \{p_j | t_j \in (t_{i-1}, t_i]\} - \min \{p_j | t_j \in (t_{i-1}, t_i]\}, \\ \text{NT}_i &= \# \{t_j | t_j \in (t_{i-1}, t_i]\}, \\ \text{Vol}_i &= \text{NT}_i^{-1} \sum_{t_j \in (t_{i-1}, t_i]} s_j, \end{aligned} \quad (16)$$

for $i = 1, \dots, n$, where $\#$ counts the elements of the set $\{\cdot\}$. Note, that other proxies for price variations as, e.g., the 10 min realized volatility or the squared returns, can replace the high-low range.

To remove the U-shaped daily seasonal pattern provided by the variables defined above, the individual seasonal components are approximated by fitting cubic splines and each series is divided by the respective estimated seasonal factor. Then, model (8) with mean (9) is calibrated to the process, where $A(L)$ and $B(L)$ are restricted to be diagonal and to the first lag. The infinite sums of the mean equations of the FIMEMs are truncated to 400 lagged coefficients, i.e., $\sum_{l_j=0}^{400} \pi_{lj} L^{lj}$, since the parameters ξ_j are almost unaffected by including additional π_{lj} 's, $j = 1, \dots, d$. Despite these restrictions, the estimated models produce uncorrelated residuals. Figure 1 presents scatterplots of the filtered residuals. The lower diagonal elements of Figure

1 do not reveal elliptical dependencies, thus the Gaussian copula is inappropriate in this case. In the following, we prefer an approximation of the dependence structure by the hierarchical or simple Archimedean Gumbel copula, since the bivariate contour plots indicate almost the same dependencies as the underlying scatterplots.

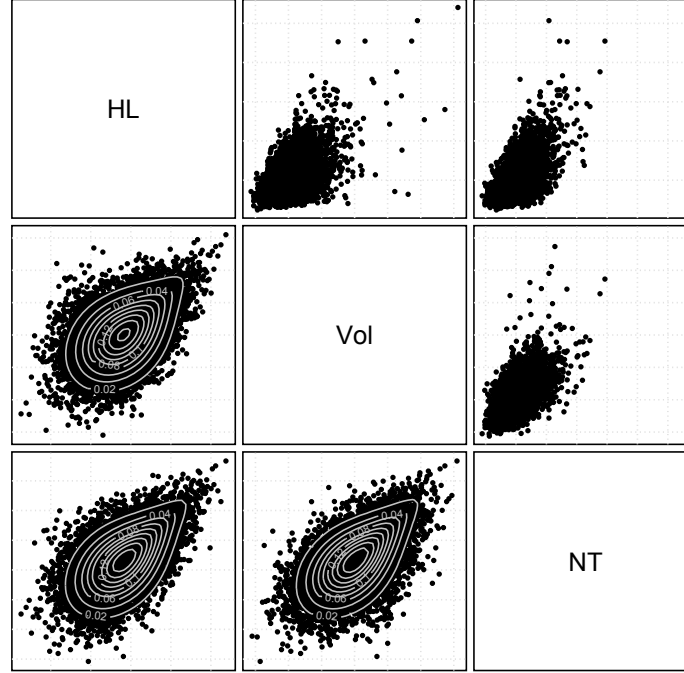


Fig. 1 The upper diagonal elements show the pairwise dependence between the filtered residuals. The lower diagonal elements present the values of the standard normal quantile applied to the values of the empirical distribution functions. Scales of the axes are not presented as they differ slightly. The origins of the coordinate planes of the upper diagonal elements correspond to zero.

The approach proposed in Section 3 considers only one single interval I , whose subintervals, defined through the set of possible change points \mathcal{T}_I , are tested for homogeneity. This method turns out to be time-varying, when it is applied as a sequential testing procedure. For this purpose, define the set \mathcal{I} , which contains the geometrically growing sequence of nested interval-candidates $I_0 \subset I_1 \subset \dots \subset I_k \subset \dots \subset I_K$, with $I_k = [i_0 - m_k, i_0]$, reference point i_0 , geometric grid $m_k = \lceil 1.25^k m_0 \rceil$, and the sets of possible change points $\mathcal{T}_{I_k} = [t_0 - m_{k-1}, t_0 - m_{k-2}]$ for all $I_k \in \mathcal{I}$. $[x]$ means the integer part of x and $m_0 = 40$. If the null hypothesis of constant dependence is not rejected for interval I_k , the interval length is extended and interval I_{k+1} is tested for homogeneity. This procedure is continued until a change point is identified or the largest interval I_K is accepted as interval of homogeneity. If a change point is detected at $k+1$, the local adaptive estimates are given by $\hat{\theta} = \hat{\theta}_k$, $\hat{s} = \hat{s}_k$, where $\hat{\theta}_k, \hat{s}_k$ denote the ML-estimates from Section 2. While other time-varying methods

permit only the parameter(s) to vary over time, the structure of this time-varying HAC may change as well.

Based on the Gumbel family, we apply the LCP procedure to the filtered residuals, because an application of the LCP procedure to the full VMEM is cumbersome due to the large number of parameters. The first panel of Figure 2 shows the changing HAC-structure estimated for an accepted interval of homogeneity, whose length is shown in the fourth panel. The two thick solid lines (grey and black) in the second panel present the time-varying parameters in terms of Kendall's $\hat{\tau}$. For the relationship between bivariate Archimedean generators and Kendall's τ see [10]. Based on these results, we propose to re-estimate the parameters of the VMEM's scale function μ_i for at least three intervals separated by the dashed vertical lines, using full ML with fixed copula parameters. The first interval ending in the middle of March can be clearly identified, as the structure is constant and the estimates of Kendall's τ exhibit a certain distance. The HAC for this interval is given by $s_{HAC}^1 = ((NT \text{ Vol})_{1.69} HL)_{1.55}$ and the simple AC by $s_{AC}^1 = (NT \text{ Vol HL})_{1.58}$, where the subscript is related to $\hat{\theta}$. The second interval is characterized by an alternating structure, while the values of Kendall's $\hat{\tau}$ can almost be distinguished. This makes it, however, difficult, to decide, whether a HAC or a simple AC should be used for re-estimating the VMEM. In general, the corresponding HAC, $s_{HAC}^2 = ((NT \text{ Vol})_{1.63} HL)_{1.40}$, and AC, $s_{AC}^2 = (NT \text{ Vol HL})_{1.45}$, indicate a weaker dependence than the fitted copulas of the first interval. In the third interval beginning in June, the underlying copula corresponds with high probability to a simple AC, since the structure changes frequently and both parameters are very close to each other. The HAC of this interval, $s_{HAC}^3 = ((NT \text{ HL})_{1.52} \text{ Vol})_{1.40}$, shows a different structure and the AC, $s_{AC}^3 = (NT \text{ Vol HL})_{1.42}$, a weaker dependence than the calibrated copulas of the first and second interval. We admit, at this point, that shorter interval specifications are possible, as the method provides a sensitive picture of the time-varying dependence. Note, that shorter time intervals are accompanied with less data and therefore, imply a loss in efficiency. The estimated HAC based on the entire sample is given by $s_{HAC} = ((\text{Vol NT})_{1.56} HL)_{1.41}$ and the respective simple AC by $s_{AC} = (NT \text{ HL Vol})_{1.45}$. We investigated the time-varying dependence for a few of other stocks and found similar results.

The third and fourth panels illustrate the performance of the LCP procedure. As proposed in Section 3, the LR test statistic measures the stability of the fitted model. Therefore, the length of the accepted intervals increase continuously in periods of a stable fit, whereas the interval length is typically short if the ML process is volatile. The dynamic of the ML process is presented in the third picture and allows to reproduce this relationship. The ML process exhibits a higher volatility in the last two months of the observed sample. This implies shorter intervals, for which the hypotheses of homogeneity are accepted, since the LR test statistics are smaller. [11] illustrates in a simulation study, that the procedure detects dependence changes with a short delay and [24] investigates the quality of the local adaptive estimators. A simple alternative approach is the rolling window method, which also allows for time-varying parameters but detects changes in the dependence with a larger delay.

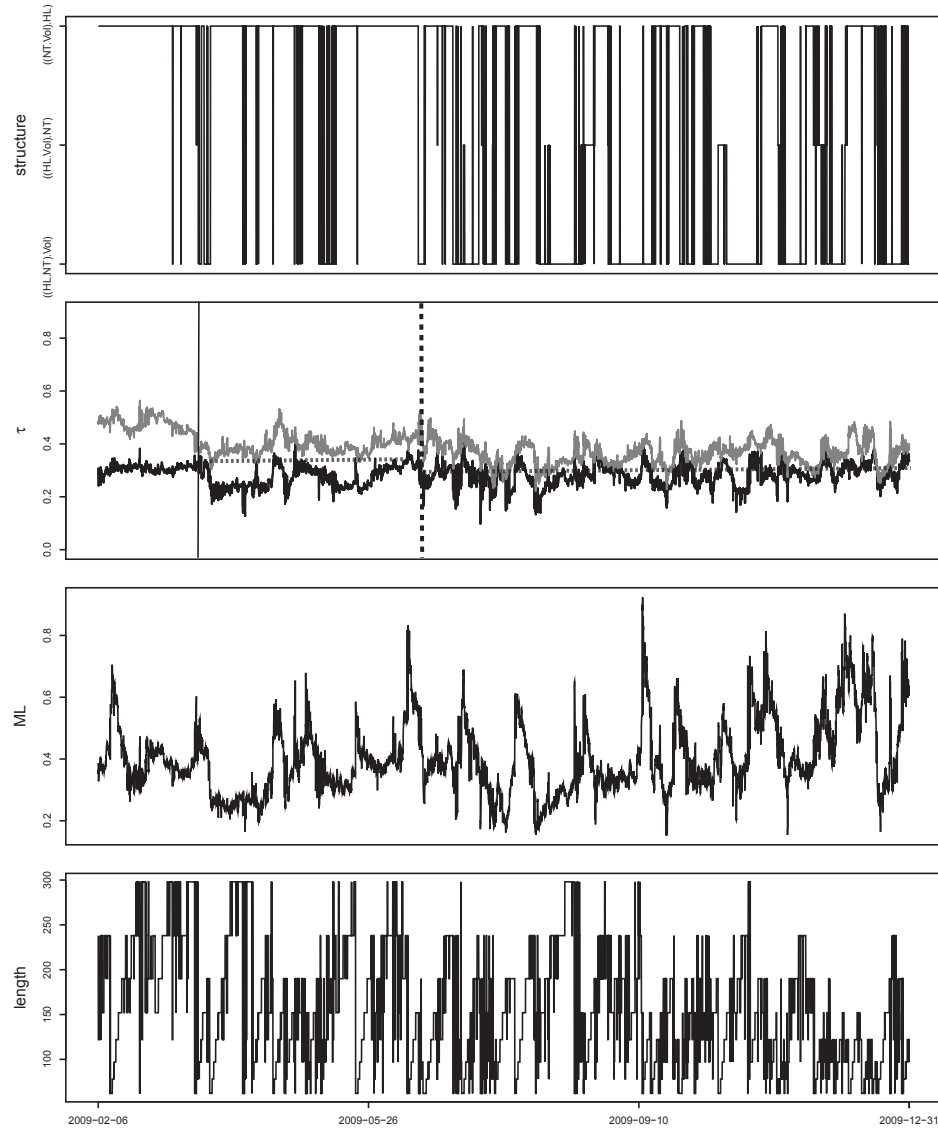


Fig. 2 Results of the LCP-procedure of AAPL. The first panel shows changes in the structure, the second the estimates of Kendall's τ and the third variations of the ML process for the intervals of homogeneity, whose varying length is presented in the lower panel.

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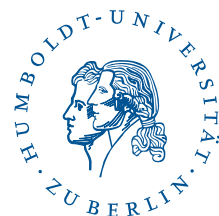
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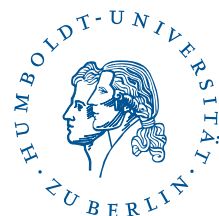
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